Trade-Off? What Trade-Off: Informative Prices without Illiquidity*

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Abstract

Private information production in financial markets enhances asset price informativeness, aiding efficient decision-making. Investors pay for information to profit from trading, creating a trade-off between market informativeness and illiquidity costs. Using a mechanism design approach, we show price informativeness can be achieved without illiquidity, at a cost equal to producing information. This mechanism incentivizes efficient information production, avoiding the inefficiency of profits obtained at the expense of less informed investors.

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1 Introduction

An important role of financial markets is to produce information about asset payoffs. This information can then be used by decision makers (e.g., firms' managers) for making more efficient investment decisions, thereby increasing firm value. However, investors' incentives to produce information derives from the profits that they can make at the expense of less informed investors (see, e.g., Grossman and Stiglitz, 1980). This happens because of the prevalence of pooling equilbria that necessarily arise as a result of chosen mechanisms. Thus, asymmetric information makes financial markets less liquid, which lowers asset values. Consequently, there is a trade-off between the benefits of more informative prices and market liquidity.

Financial markets should therefore be designed to best solve this trade-off, that is, to maximize price informativeness while minimizing illiquidity due to adverse selection costs. In this paper, we use a mechanism design approach to study this question. For concreteness, we consider an entrepreneur (the "issuer") with one asset. The payoff of this asset can be high or low and the entrepreneur does not have the expertise to discover what is the exact realization of the payoff. To do so, he can sell a fraction of the asset to investors who have the ability to discover its payoff by collecting additional data. Doing so is costly and uncertain: with some probability, no information can be discovered about the payoff. The entrepreneurs' expected profit from the sale of the asset is equal to the proceeds from the sale plus a gain proportional to the reduction in the uncertainty on the asset payoff (e.g., this could be the gain derived from investing in other projects whose payoffs are correlated with the asset payoff). The entrepreneur chooses to sell a fraction of the asset if the maximal value of this expected profit exceeds the expected payoff of the asset (that is, the entrepreneur's outside option is to do nothing).

The entrepreneur's objective is to design the issue to maximize her expected profit. As all investors are rational and competitive, all costs borne by investors are ultimately passed back to the entrepreneur. Thus, the entrepreneur's expected proceeds from the issue cannot exceed the expected payoff of the asset net of information acquisition costs borne by investors. However, they can be less than this upper bound.

Indeed, to incentivize some investors to pay the information cost, the issuer must either pay them directly or let them earn profits at the expense of investors who do not buy information (e.g., as in Rock (1986) or Holmström and Tirole (1993)). In the first case, the issuer faces an agency problem (investors may misreport the information that they obtain or not pay the cost of information acquisition). To satisfy incentive constraints, the issuer might have to leave informational rents to investors. In the second case, uninformed investors will pass expected losses ("adverse selection costs") to the issuer by discounting the price at which they buy the asset. In sum, the entrepreneur faces both agency and adverse selection costs and seeks to design the issue to minimize these costs.

In this setting, we show that there is a mechanism that makes the entrepreneur's expected profit arbitrarily close to the maximum expected profit she can expect (the one obtained in the absence of agency and adverse selection costs). The mechanism has two stages. In the first stage, investors are sequentially offered the possibility to buy two derivatives securities, one that pays only if the asset payoff is high and one that pays only if it is low. If an investor refuses to participate, she retains the possibility to participate to stage 2. The entrepreneur optimally decides when to stop stage 1 and move to the second stage in which he sells the issue at a fixed price, after announcing publicly the outcome of the first stage (that is, the number of investors who participated to this stage, the number of derivatives sold and the type of derivatives traded). The entrepreneur chooses (i) derivatives' prices in stage 1, (ii) the payoff of each derivative, (iii) the number of investors participating to stage 1 (when to stop), (iv) the investors who can participate to stage 2 (he can exclude some of the investors who participated to stage 1) and (v) the price of the issue.

We show that the entrepreneur can design the derivatives (their payoff and price) in such a way that an investor who participates to stage 1 finds optimal to (i) produce information and (ii) select the derivative security that truthfully reveals the asset payoff if she learned this payoff. Moreover, if an investor does not discover information, she optimally abstains from buying or selling a derivative. Given these choices, the first investor who buys a security fully reveals the payoff of the asset. Thus, to minimize information acquisition costs, it is optimal for the issuer to stop stage 1 as soon as one investor trades a derivative security and moves to stage 2. In this stage, the entrepreneur sells the asset at a price equal to its payoff.

As stage 1 takes place sequentially, the entrepreneur and investors become increasingly pessimistic about whether information about the asset payoff exists as the number of investors contacted to participate to stage 1 increases. Intuitively, this makes the cost of incentivizing information production higher over time because investors increasingly expect to pay the search cost without discovering information. Thus, to be incentivized to pay the information acquisition cost, investors must expect an increasingly higher payoff from the derivatives, which is costly to the issuer. As a result, unless information is available with probability 1, the entrepreneur optimally stops stage 1 at some point even if no investor bought a derivative. In this case, no information is produced in stage 1. Anticipating this outcome, some investors might refuse to participate to stage 1 and acquire information before participating to stage 2. However, we show that the entrepreneur can optimally avoid this outcome by pushing further the moment at which she stops stage 1.[1] In this way, the entrepreneur avoids underpricing the issue to induce uninformed investors to participate.

When information is produced in stage 1, the entrepreneur realizes the gains of obtaining information without paying illiquidity costs due to the risk of adverse selection for uninformed investors. When information is not produced, the entrepreneur does not obtain gains associated with information production but she can issue shares at the average payoff of the asset, that is, without paying illiquidity costs due to informed investors participating to the issue.[2] For these reasons, the expected profit of the entrepreneur with this mechanism is arbitrarily close to the one she can obtain when there are no agency and adverse selection frictions. The entrepreneur just needs to compensate investors who search for information.

Our paper relates to two strands of the literature. First, it relates to the literature on the informational benefits of financial markets for firms. These benefits can stem

[1]Intuitively, the entrepreneur delays the closure of stage 1 until the likelihood that information exists is so small that the expected profit from informed trading in stage 2 is less than the information cost.

[2]In this case, the entrepreneur is indifferent between issuing shares or not.

from the use of information in stock prices for contracting (see, for instance, Holmström and Tirole (1993)) or for making investment decisions (e.g., Edmans, Goldstein, and Jiang (2015); see also Bond, Edmans, and Goldstein (2012) and Goldstein (2022) for surveys)). In some papers in this literature, firms choose the fraction of shares to issue facing a trade-off between the benefits of informative prices (the gain associated with using the information in prices) and illiquidity costs (see, for instance, Holmström and Tirole (1993), Subrahmanyam and Titman (1999), Faure-Grimaud and Gromb (2004) or Foucault and Gehrig (2008). However, we are not aware of papers that seek to analyze how firms should optimally design the sale of shares to investors when they face this trade-off.

The literature on initial price offerings has analyzed the sale of shares to the public using a mechanism design approach (see, for instance, Beneviste and Wilhelm (1990) or Biais, Bossaerts, and Rochet (2002)). However, in most the literature on this topic, informed investors are supposed to be exogenously endowed with private information and firms do not derive gains from the information produced during their price offering. Exceptions are Sherman (2005) and Sherman and Titman (2002) and our framework is closely related to their modeling approach (in particular, the information structure is identical). However, they do not consider the possibility of using a sequential mechanism with two stages as we do. As discussed at the end of our paper, this possibility makes the issuer better off (that is, the mechanism considered in our paper dominates that considered in Sherman (2005) and Sherman and Titman (2002)).

2 The Problem: Illiquidity versus Informativeness under Asymmetric Information

In this section we first illustrate the tension between illiquidity and informativeness of the trading process using a standard modeling approach for the sale of a risky asset. Importantly, we assume that some investors have private information about the payoff of the asset. This information is exogenous. Hence, to obtain information, the issuer just needs to incentivize these investors to reveal their information, not to produce it. We relax this assumption in the next section, which constitutes the core of our contribution.

The model is as follows. One agent owns $Q+N$ shares that are claims on the payoff of a risky asset of which it wishes to sell Q shares. The payoff of the asset (per share) is v_H with probability μ or v_L with probability $(1-\mu)$. There are $H+I$ potential buyers (henceforth investors), where I is the number of investors with information about the payoff of the asset. These investors perfectly know the realization of v while the remaining investors only know the distribution of v. Each investor can buy only up to one share and $Q < H$. Thus, the asset seller does not need participation of informed investors to execute her trade. The seller cannot observe who is informed and who is not (or cannot price discriminate based on investors' types).

There are several possible interpretations of this set-up. First, one can interpret the asset seller as a firm selling shares to the public in an initial price offering (IPO). This is our leading example and, for this reason, we refer to the seller as the issuer. Alternatively, one can see the seller as an entrepreneur selling a fraction of its stake to venture capitalists or business angels.

One can consider several ways to organize the sale of the asset. We first contrast two methods. The first is such that the issuance process fully reveals the payoff the asset but it results in underpricing due to adverse selection. The second is such that there is no underpricing because it excludes participation from informed investors. However, as a result, the issuance process provides no information about the asset payoff. These are just manifestations of the trade-off between illiquidity, due to adverse selection, and illiquidity.

In the first method, the issuer sets a price p_{issue} and investors decide whether they want to participate or not at this price. If there is excess demand, the issuer allocates shares pro-rata to each investor willing to buy one share at p_{issue}. This is a fixed price offering, as in the Rock (1986)'s model. For the issue to succeed, the issuer must guarantee the participation of uninformed investors. Suppose that $v_L < p_{issue} < v_H$ and consider a situation in which it is optimal for each uninformed investor to buy one share at this price. At this price, each informed investor finds it optimal to buy one share if $v = v_H$ and to abstain otherwise. Thus, when $v = v_H$, each uninformed investor only receives $q_u(v_H) = \frac{Q}{H+I}$ shares (pro-rata rationing), while when $v = v_L$ each uninformed investor receives $q_u(v_L) = \frac{Q}{H}$. Thus, the expected profit of uninformed investors is:

$$E(q_u(v)(v - p_{issue})) = \mu q_u(v_H)(v_H - p_{issue}) + (1-\mu)q_u(v_L)(v_L - p_{issue}). \qquad (1)$$

To guarantee the participation of uninformed investors (which is necessary for the issue to succeed) and maximize the proceeds of the issue, the issuer must choose the largest price such that $E(q_u(v)(v - p_{issue})) \geq 0$, which is the price solving $E(q_u(v)(v - p_{issue})) = 0$. Thus, the issuing price is:

$$p^*_{issue} = \beta v_H + (1-\beta)v_L,$$

with $\beta = \frac{\mu H}{H+(1-\mu)I}$. As $I > 0$, $\beta < \mu$ and therefore $p_{issue} < E(v)$.

Thus, the issue must be underpriced for it to succeed. Note that in this case, the issuing price does not reveal information about v since it is identical whether informed investors participate or not in the issue. However, total demand in the issue fully reveals the asset payoff. Thus, the trading process fully reveals investors' private information about the payoff of the asset. However, this information is obtained by the issuer at the cost of underpricing (illiquidity).

Now consider a more complex method for issuance. With this method, the issuer is allowed to make the issuance price contingent on demand. Specifically, let D be the total demand in the issue and consider the following price schedule posted by the issuer:

$$p_{issue} = \begin{cases} v_H + \epsilon, & \text{if} \quad D > H \quad \text{and} \quad \epsilon > 0, \\ E(v), & \text{if} \quad D \leq H. \end{cases} \qquad (2)$$

In this case, the following decisions for investors form a Nash equilibrium: (i) informed investors do not participate, (ii) uninformed investors offer to buy 1 share. To see that this is an equilibrium, consider informed investors first. As the issuing price is always strictly larger than v_L, it is never optimal for an informed investor to buy when $v = v_L$. When $v = v_H$, if an informed investor buys, she expects total demand to exceed H and therefore the price to be $v_H + \epsilon$. Thus, not participating is a best response to the issuer's price schedule and uninformed investors' strategy. Given that informed investors never participate, uninformed investors anticipate that they will receive $q_u = q_u(v_H) = q_u(v_L) = \frac{Q}{H}$ whether $v = v_H$ or $v = v_L$ and that total demand will always be $D = H$. Thus, their expected profit is:

$$E(q_u(v)(v - p_{issue})) = q_u \left(\mu(v_H - p_{issue}) + (1 - \mu)(v_L - p_{issue})\right) = 0.$$

Thus, uninformed investors are indifferent between participating or not, and participation is therefore a best response to the issuer's price schedule. The issuer cannot do better intuitively since any price larger than p_{issue} cannot satisfy uninformed investors' participation constraint. Thus, this equilibrium maximizes the expected proceeds for the issuer. However, ex-post, the issue price and total demand are completely uninformative since they are identical whether the payoff of the asset is high or low. This issuance method avoids underpricing (illiquidity) by removing adverse selection, at the cost of informativeness. This is again a manifestation of the standard trade-off.

We refer to this second mechanism as the "no-informed trading" mechanism. It is optimal for the issuer (it maximizes the expected proceeds from the sale of the asset) if the latter does not derive any benefit from the information produced during the issuance process. However, if it does (e.g., it could use the information for making new investments) and if this benefit is large enough, the first method can dominate the second. However, we show below that there is another mechanism that (i) avoids underpricing and (ii) is fully revealing. Thus, this mechanism eliminates the trade-off between illiquidity and informativeness and dominates the two previous methods. We refer to this mechanism as the "divide and conquer" mechanism.

In this mechanism, the issuance process is organized in two stages. In the first stage, investors are contacted sequentially and offered the possibility to buy 2 derivative contracts from the issuer whose payoffs are contingent on the realization of the fundamental value v, when this is finally observed. The first contract, labelled C_L

pays $F + \epsilon$ if $v = v_L$ and zero otherwise, where $F, \epsilon > 0$ are some predetermined positive values. The second contract, labelled C_H pays $F + \epsilon$ if $v = v_H$ and zero otherwise. All derivative contracts expire right after the end of the trading round after the fundamental value of the asset is observed. The price of each contract is F. The first stage stops when one investor has decided to buy one of the contract or when all investors have been contacted.

The issuer reveals the outcome of this stage to all investors and then move to the second stage. In the second stage the underwriter allocates the Q shares among the remaining $H + I - 1$ investors at $p_{issue} = v_L$ if the investor participated in the first stage has chosen C_L and $p_{issue} = v_H$ if the investor participated in the first stage has chosen C_H. If no investor participates to the first stage then the underwriter cancels the issue and no allocations is done (this never happens in equilibrium).

We say that this mechanism induces full revelation if (i) only informed investors buy in stage 1 and (ii) an informed investor selects contract C_w when she observes that $v = v_w$ for $w \in \{L, H\}$.

Proposition 1. *If the issuer chooses $F > \max\{\frac{(1-\mu)}{\mu}, \frac{(\mu)}{(1-\mu)}\}\epsilon$ and $\epsilon > 0$, the mechanism induces full revelation and the expected proceeds per share from the asset sale are $E(v) - \epsilon$. In this case, the Nash equilibrium of the issuance process is that (i) uninformed investor do not trade in stage 1, (ii) the first informed investor contacted by the issuer in stage 1 chooses contract C_w when she observes that $v = v_w$ for $w \in \{L, H\}$ and (iii) the issuing price in the second stage is $p_{issue} = v$ so that investors participating in the second stage are indifferent between buying the asset or not.*

With this divide and conquer mechanism, the expected proceeds from the sale of the asset, $E(v) - \epsilon$, are arbitrarily close to the maximum expected proceeds, $E(v)$ because ϵ (the net payoff of the derivative contracts) can be arbitrarily small (it just needs to be strictly positive). Thus, the mechanism optimally solves the trade-off between illiquidity and informativeness in the framework considered so far. Intuitively, the mechanism separates the problem of incentivizing informed investors to reveal their private information from the problem of incentivizing uninformed investors to participate to the issue. In the two previous methods, these problems are bundled. The divide and conquer mechanism separates them and creates competition between informed investors to minimize the cost of information revelation for the issuer. Intuitively, this cost cannot be less than the cost of information production, as otherwise informed investors would not participate. However, so far, we assume that investors bear no information production cost (they are exogenously endowed with information). Thus, intuitively, by inducing competition among informed investors, the issuer can drive the cost of information revelation, ϵ, to almost zero. The condition $\epsilon > 0$ is just to make an informed investor strictly better off participating to the first stage.

In the next section, we show that this insight still obtains when information production is endogenous. In this case, the divide and conquer mechanism must not only induce investors with information to truthfully reveal their information via their choice in stage 1 but also induce them to produce information. We discuss the robustness of the mechanism to more general environments in Section 6. We think that the divide and conquer mechanism offers an interesting benchmark for assessing frictions in real-world financial markets. This mechanism solves the trade-off between informativeness and illiquidity. Hence, if it is not used, it must be that other frictions make it impractical or dominated by other mechanisms. Identifying reasons why divide and conquer mechanisms are not used more is then the question.

3 Costly Information Production

In this section, we now consider the case in which information production is endogenous. This case is more complex because the mechanism that is used by the issuer must incentivize investors both to reveal their information if they have some and to produce information. To consider this issue, we modify the previous framework as follows. As before, there are $H + I$ investors and each investor can buy only up to one share and H and I are large relative to Q. Only I investors have the ability to produce information about the asset. However, at the beginning of the asset sale, these investors have not yet information and must pay a cost to produce it. We denote by $\mathcal{I}$ (resp., $\mathcal{H}$) the set of investors who (resp., don't) have the ability to produce information.

Information production is as follows. There is a probability $\pi \in (0, 1)$ a probability that there information about the fundamental value of the firm. To produce information about v, an investor must pay a cost c without knowing whether information is available or not. After paying the cost c, if information exists, the investor is succesful, i.e., learns v perfectly with probability $\phi \in (0, 1)$.Otherwise, that is, if the investor is unsuccessful or if information does not exist, the investor remains uninformed. Thus, the likelihood that an investor fails in producing information is $(1-\phi)\pi + (1-\pi) = (1-\phi\pi)$. Importantly failure to produce information does not imply that information does not exist since $\phi < 1$. To ensure that the information cost is not prohibitively high, we assume that $\frac{c}{\pi\phi} < Qv_L$ (that is, the expected cost of information acquisition is smaller than the value of the firm in bad state).

If instead the investor does not search for information, she remains uninformed and expects the value of the asset to be $v_U = E(v) = \mu v_H + (1-\mu)v_L$ (that is, she has access to the same information as the issuer and other uninformed investors). We assume that the issuer cannot acquire information.[3] This is a natural assumption since we want to analyze the trade-off between informativeness and illiquidity from the asset seller's viewpoint. If the asset seller could pay the cost of information, she would not need to incentivize information production in the first place.

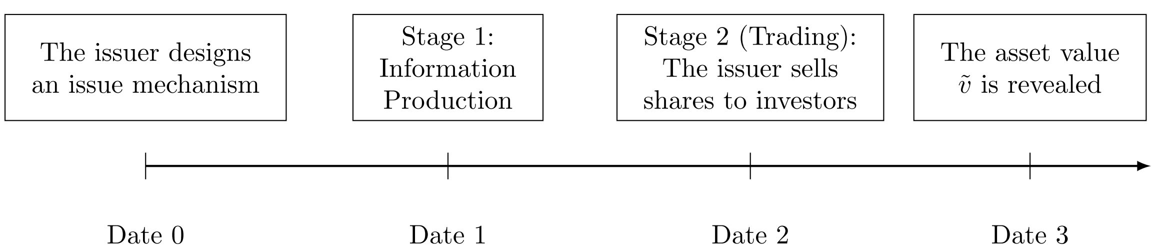

Figure 1. Timing of the model

Figure 1 presents the timing of the model. At date 0, the seller of the asset

[3]This does not mean that the issuer has no information. Indeed, one can assume that the issuer first collects information and arrives to an estimate of $E(v)$ for the firm. It just means that the cost of collecting incremental information is too high for the issuer.

designs and announces the mechanism that it will use to sell shares to investors. As explained below, this choice is made to maximize the proceeds from the sale and the "informativeness" of the sale. The mechanism is similar to the divide and conquer mechanism presented in the previous section. In the first stage ("information production"), the seller contacts investors sequentially and ask them to report their information about the asset. In the second stage ("trading"), the seller proceeds to the sale of the asset. In contrast to the divide and conquer mechanism presented before, in stage 1, investors directly report their information (or absence thereof) and receive a transfer from the issuer rather than pick a derivative. This difference is not important: The use of derivatives is just a way to implement the direct mechanism considered here. As explained in subsequent sections, the more substantial difference is that the mechanism must make sure that investor who reports information have indeed paid the cost of information production since their effort is not observable. Last, we assume that at some point in the future the fundamental value of the asset is realized, whether information production took place or not. This assumption plays a role in the design of the incentive mechanism considered in Section 5.[4]

We denote the price at which the asset is sold in stage 2 by p_{issue} and we denote by p_2 the price of the asset at date 2, just after stage 2. This price will depend on the information publicly available after stage 2 and therefore be different from the price at which the asset is sold at stage 2. For instance, if information is produced and fully revealed to market participants, p_2 will be v_H or v_L. However, this might not be the case for the price at which the asset is sold in stage 2, giving the rise to the possibility of underpricing or overpricing.

The seller's utility depends on her proceeds from the sale of the asset and the informativeness of the sale. Her proceeds are equal to $Qp_{issue} - C_{issue}$, where C_{issue} are total monetary transfers to investors participating to stage 1 (they can be zero or even negative; see below). The informativeness of the sale is measured by the residual uncertainty about the payoff of the asset after observing the outcome of stages 1 and 2. We denote the seller's information set at the end of stage 2 by Ω_2. It contains, for

[4] This is also the case in the divide and conquer mechanism considered in Proposition 1 since the payoff of the derivatives depends on the realization of the fundamental value.

instance, the reports in stage 1 and the price of the asset after stage 2, p_2. Residual uncertainty for a given realization of Ω_2 is measured by $Var(v \mid \Omega_2)$. The realized utility of the issuer is after the sale of the asset is therefore:

$$\Pi(p_{issue}, C_{issue}, \Omega_2)) = Qp_{issue} - C_{issue} - \gamma Var(v \mid \Omega_2), \tag{3}$$

where γ measures the utility gain for the seller from a marginal decrease in uncertainty about v after the sale of the asset. Parameter γ measures the importance of the informativeness of the mechanism for the seller. If $\gamma = 0$, the seller does not care about informativeness and, as we shall, see in this case she will organize the issue so that no information is produced. In this case, the illiquidity-informativeness trade-offis moot since information has no value. Thus, the more interesting case is $\gamma > 0$. As explained below, the seller designs the mechanism for selling the asset at date 0 to maximize $E(\Pi(p_{issue}, C_{issue}, \Omega_2)))$, the expected value of her realized utility after the issue.

3.1 Benchmark: Information Production is Observable

As a benchmark, we first consider the case in which the issuer can observe whether a given investor has the ability to produce information or not and that investors always truthfully report the outcome of their search for information. Moreover, we assume that the issuer can exclude informed investors from stage 2 (e.g., by using the no-informed mechanism described in Section 2). Thus, in this benchmark, we consider the case in which there is no moral hazard in stage 1 and no adverse selection in stage 2. In this case, the issuer's problem is to obtain information at the lowest possible expected cost.

In this case, the issuer faces no incentives compatibility constraints (investors don't need to be incentivized to report truthfully what they know). It must still design the issuing mechanism to guarantee participation by investors to each stage. This means in particular that, in stage 1, the issuer must compensate investors for their information production cost (as otherwise they would not produce information). Moreover, in stage 2, the issuer cannot sell the asset at a price larger than its expected payoff conditional on the information produced during stage 1, as otherwise investors would not buy shares in Stage 2. Given this, the largest expected proceeds that the issuer can achieve are equal to $QE(v)$ minus the expected information acquisition costs for investors in stage 1. We show below that this is indeed the case. Moreover, the maximum expected utility achieved by the issuer in this case is an upper bound for its expected utility in the case in which the issuer does not observe whether investors acquire information because in this case the issuer face additional incentives compatibility constraints (see Section 5).

In stage 1, the issuer contacts investors with the ability to produce information sequentially, that is, investors in $\mathcal{I}$.[5] Each contacted investor optimally chooses to produce information or not and reports the outcome of her search to the issuer. If she chooses to produce information, the investor pays the information acquisition cost, observes the outcome of her search for information and finally reports a message $s \in \{H, L, U\}$ to the issuer, where $s = H$ means that the investor has discovered $v = v_H$, $s = L$ means that the investor has discovered $v = v_L$ and $s = U$ means that the investor has found nothing. To compensate the i^{th} investor, the issuer pays a fee f_{i,s_i} which can depend on the investor's report (s_i) and his position (i) in the queue of contacted investors.[6] If the investor chooses not to produce information, she receives no reward.

Importantly, this process brings information about whether information about v is available or not. Indeed, since the information is not present with certainty $(\pi < 1)$, investors (as well as the issuer) update their beliefs about availability of information after every unsuccessful round of information acquisition. The probability that there is information available about the payoff of the asset conditional on observing $i - 1$ uninformative signals in a row is:

$$\pi_i = \frac{(1-\phi)^{i-1}\pi}{(1-\phi)^{i-1}\pi + (1-\pi)}. \tag{4}$$

[5] Contacting investors in $\mathcal{H}$ is useless for the issuer since they cannot help the issuer to obtain information.

[6] We assume that investors know their position in the queue.

Observe that $\pi_1 = \pi$ and that π_i decreases with i. Thus, investors participating to stage 1 and the issuer becomes increasingly more pessimistic about the possibility of finding information as the length of stage 1 increases.

The i^{th} investor produces information if her expected reward exceeds the cost of information production, that is, if:

$$\pi_i \left[\phi\mu f_{i,H} + \phi(1-\mu)f_{i,L} + (1-\phi)f_{i,U}\right] + (1-\pi_i)f_{i,U} \geq c, \quad i \in \{1, ..., \tau\}, \tag{5}$$

This equation is the participation constraint of the i^{th} contacted investor in Stage 1. The L.H.S is the expected fee received by the investor producing information and the R.H.S is the cost of producing information. Thus, eq.(5) is the participation constraint of the i^{th} investor.

As π_i decreases over time when $\pi < 1$, there is a information production round K^* after which contacting subsequent investors to obtain information is not optimal. Thus, when the K^*th investor fails to find information, the issuer's expected utility is

$$QE(p_{issue}) - c - \gamma\mu(1-\mu)(v_H - v_L)^2.$$

If instead, the issuer contacts one extra investor and then moves to stage 2, his expected utility is:

$$QE(p_{issue}) - \gamma(1 - \pi_{K^*+1}\phi))\mu(1-\mu)(v_H - v_L)^2,$$

because $\Pr(\Omega_1 = U) = (1 - \pi_{K^*+1}\phi)$ in this case. By definition of K^*, this course of action must be dominated by moving to stage 2 not optimal if and only if $\gamma\pi_{K^*+1}\phi\mu(1-\mu)(v_H - v_L)^2 > c$. This implies that $K^* = K_{max}$.

Once an investor has found information, there is no incentive for the issuer to keep contacting investors in $\mathcal{I}$ since uncertainty about v has been fully resolved and the outcome of stage 1 is publicly announced. Thus, stage 2 should optimally stop when one investor reports $s = H$ or $s = L$. The issuer could also optimally stop when $s = u$ after many trials because inducing investors to produce information becomes increasingly costly as i increases when $\pi < 1$ (see the participation constraint eq.(5)). Thus, we let K be the total number of contacted investors in stage 1 be another choice variable for the issuer. This number can be smaller or larger than I because one informed investor can be asked repeatedly to produce information. We denote by τ_{stop} the number of rounds in stage 1. This number is the minimum of K and the first time at which an investor finds information. For a given realization of τ_{stop}, the total cost of stage 1 for the issuer is therefore:

$$C_{issue} = \sum_{i=0}^{i=\tau_{stop}} f_{i,s_i} = (\tau_{stop} - 1) f_{i,U} + f_{\tau_{stop}, s_{\tau_{stop}}}. \tag{6}$$

Observe that C_{issue} is random because the stopping time for stage 1 is random since whether investors discover or not information in stage 1 is random.

After stage 1 is completed, the issuer announces the outcome of this round and sets a price p_{issue} for the issue. The outcome, Ω_1 is H if one investor has reported $s = H$, L if one investor has reported $s = L$ and U otherwise. As we assume that an investor with information cannot participate to stage 2, we must have $p_{issue} \leq E(v \mid \Omega_1)$ to guarantee participation of uninformed investors to stage 2. Last, as the trading process in stage 2 is uninformative (since no informed investors participate to this stage), $\Omega_2 = \Omega_1$. Thus,

$$\mathrm{E}(Var(v \mid \Omega_2)) = \Pr(\Omega_1 = U)\mu(1-\mu)(v_H - v_L)^2, \tag{7}$$

where $\Pr(\Omega_1 = U)$ is the probability that no information is produced during stage 1.

Thus, for a given design of stages 1 and 2, we deduce from eq.(3) and eq.(7) that the expected utility of the issuer is:

$$\Pi(p_{issue}, \{f_{i,s_i}\}, K\}) = QE(p_{issue}) - E(C_{issue}) - \gamma \Pr(\Omega_1 = U)\mu(1-\mu)(v_H - v_L)^2. \tag{8}$$

At date 0, the issuer chooses $\{p_{issue}, \{f_{i,s_i}\}, K\}$ to maximize her expected utility, under the constraints that investors participate to stages 1 and 2. Thus, she solves the following problem:

$$\Pi_{bench} = \max_{\{p_{issue}, \{f_{i,s_i}\}, K\}\}} \Pi(p_{issue}, \{f_i\}), \tag{9}$$

subject to the participation constraints:

$$\pi_i \left[\phi\mu f_{i,H} + \phi(1-\mu) f_{i,L} + (1-\phi) f_{i,U}\right] + (1-\pi_i) f_{i,U} \geq c, \quad i \in \{1, ..., \tau\}, \tag{10}$$

$$p_{issue}(s) \leq \mathrm{E}(v \mid \Omega_1) \tag{11}$$

for every $s \in \{H, L, U\}$. Observe that K affects the expected utility of the issuer because it determines the distribution of the stopping time. One can solve the problem in two steps. First, for a given $(p_{\text{issue}}, \{f_{i,s_i}\})$, one can solve for the optimal $K^*(p_{\text{issue}}, \{f_{i,s_i}\})$. Then, in a second step, one can solve for the $\{p_{\text{issue}}, \{f_{i,s_i}\}\}$ that maximizes: $\Pi_{\text{bench}}(p_{\text{issue}}, \{f_{i,s_i}\}, K^*(p_{\text{issue}}, \{f_{i,s_i}\}))$.

Define $K_{\max}$ to be the maximal i satisfying

$$\frac{c}{\pi_i \phi} < \gamma\mu(1-\mu)(v_H - v_L)^2. \tag{12}$$

We assume that this condition holds for $i = 1$, i.e., for $\pi_i = \pi$ (the case in which it does not is discussed below) so that $K_{max} > 1$. The solution to the issuer's problem in this case is as follows.

Proposition 2. *In the benchmark case, the issuer's optimal issuance strategy is as follows.*

1. *Stopping time: the issuer stops contacting investor as soon as it obtains a positive ($s = H$) or a negative ($s = L$) report or the number of rounds exceeds $K_{\max}$;*
2. *Fees: conditional upon observing $s_{i-1} = U$, the issuer sets $f_{i,L} = f_{i,H} = \frac{c}{\phi\pi_i}$ and $f_{i,U} = 0$.*
3. *Price: the issuer sells shares to Q investors (chosen randomly) at price $p_{issue} = E(v \mid \Omega_1$.*

4. Value of objective function:

$$\begin{aligned}\Pi^*_{bench} &= M + (Q+N)E(v) - cK_{\max}(1-\pi) - \frac{c\pi(1-(1-\phi)^{K_{\max}})}{\phi} \\ &- \gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{\max}}\right)(v_H - v_L)^2. \end{aligned} \quad (13)$$

Proof. See Appendix. □

Given our assumptions, it is straightforward that the issuer should sell shares in stage 2 at $p_{issue} = E(v \mid \Omega_1)$. A lower price would leave rents to investors while at a larger price investors would not buy shares. As a result, the issuer expects to sell shares at $E(v)$.

The information produced in stage 1 is useless to increase the proceeds from the issue because there is no adverse selection in stage 1. However, it is useful to reduce uncertainty about the payoff of the asset. In designing stage 2, the issuer trades-off the benefit of reducing uncertainty with the cost of producing information.

The issuer always sets its fees for information production so that the participation constraint of each investor contacted to produce information is binding. Thus, when an issuer contacts an investor, he expects to pay c to the investor. However, the issuer's optimal fee structure is to reward the investor only if the search for information is successful. Thus, it pays the investor more than c (in fact c/ϕ) when the investor is successful in finding information and nothing otherwise.

We have assumed that the issuer contacts investors sequentially one by one in stage 2. An alternative is to contact investors by batches of M_i investors in each round i. We call this the "batched process". the next proposition that the optimal size of a batch is $M_i = 1$ for each $i \in \{1, 2, ..., K^*\}$. Thus, the process we have considered so far is the optimal way to organize information production in stage 2.

Proposition 3. *In the batched procedure, the optimal size of a batch is $M_i = 1$ in any round. Thus, the sequential procedure where the issuer contacts exactly one investor per round is optimal for the issuer.*

Proof: see Appendix

The intuition is as follows. Suppose that the issuer deviates from the previous policy by contacting $M_1 > 1$ in the first batch. In this case, the issuer must pay $M_1 c$ for sure to all investors contacted in the first batch (as each must expect a payment of c to produce information) and the likelihood that none of these investors find information is $(1 - \pi) + \pi(1 - \phi)^{M_1}$. The likelihood of this event is identical to that if investors are contacted sequentially. However, in the latter, the expected payment to investors is strictly smaller than $M_1 c$ because there is the possibility that one investor finds information before all investors are contacted, in which case the issuer optimally stops the search for information. Last, conditional on none of the M_1 investors finding information, the continuation value for the issuer is exactly the same if he contacts the M_1 investors sequentially or not. Thus, the issuer is strictly better off not contacting the M_1 first investors in a batch. The same argument can show that this is also the case at any round.

In sum, Π^*_{bench} is the largest possible expected utility for the issuer. It serves as benchmark to measure the efficiency of the various mechanisms that the issuer can use in the more complex case in which (i) the issuer does not observe investors' information acquisition decision and the signals received by informed investors and (ii) the issuer cannot prevent investors from choosing to secretly produce information when contacted to participate to stage 2. In this case, the issuer faces a moral hazard problem in stage 1 and there is adverse selection in stage 2, which may force the issuer to sell the asset at a discount, as explained in Section XXX. One may think that these frictions will reduce the expected utility that the issuer can achieve due to the trade-off between liquidity and informativeness. However, in Section, we show how the issuer can design a mechanism that makes the issuer's expected profit arbitrarily close to Π^*_{bench}. This implies that this mechanism dominates any other mechanism that the issuer could use in the context of our model (in particular that proposed by Sherman and Titman (2002) in the same environment or modification of Rock (1986) model to costly information acquisition).

4 Mechanisms

In this section we describe several competing mechanisms that can be considered as candidates to implement the first-best allocation described in the benchmark model.

4.1 Fixed price mechanism (FP)

The pooling mechanism (**FP**) is an extension of the fixed price offering of Rock (1986) where the costly information has to be endogenously acquired. In this mechanism the issuer sets a pooling price p_{issue} and let investors decide whether they want to participate or not in the issue at this price. If there is excess demand, the issuer allocates shares pro-rata to each investor willing to buy one share at p_{issue}. Those investors who have the ability to search and acquire the information endogenously decide where or not to do so.

The optimal strategies of the issuer and the investors are as follows.

Proposition 4. *Under* **FP** *mechanism:*

- *the issuer offer the issue price with underpricing $p_{issue} < E(v)$;*
- *there is a number $0 \leq K_{FP} \leq I$ of investors who participate in the information production;*
- *the issuer objective function under this strategy is:*

$$\begin{aligned} \Pi_{FP} &= M + (Q+N)E(v) - cK_{FP} \\ &- \frac{\gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{FP}}\right)(v_H - v_L)^2}{\mu\left(1-\pi+\pi(1-\phi)^{K_{FP}}\right)+(1-\mu)}. \end{aligned} \tag{14}$$

Proof: See Appendix

4.2 No-information production mechanism (NI)

Now consider another mechanism in which the issuer sets a price of $v_H + \epsilon$ if the total demand in the IPO is strictly larger than H and a price equal to $p_{issue} = E(v)$

otherwise. Using the same argument as before with this mechanism, no investor searches for information. Thus, all investors are indifferent between participating or not and the case in which just H investors participate is an equilibrium. There also equilibria in which less than H but more than Q investors participate. In this "mechanism", the issuer gets an expected utility of:

$$\Pi_{NIM} = M + (N + Q)E(v) - \gamma\mu(1 - \mu)(v_H - v_L)^2. \tag{15}$$

5 Optimal mechanism

In this section we describe the sequential mechanism (**SEQ**) that implements an allocation provided in the benchmark model.

5.1 Description of the mechanism

As described in the benchmark case, the issuance is happened in two stages. At the beginning of date 1 investors to apply for Stage 1 trading. The issuer randomly chooses one of the applicants to trade in Stage 1. Before trading, the chosen investor can (but might choose not to) attempt to acquire information. During Stage 1 the investor can either buy one of the offered by the issuer derivative contracts whose payoffs are contingent on the realization of the firm value v or decide (upon observing the information acquisition process) not to trade derivatives. If the chosen investor decides not to trade in Stage 1, that investor is excluded from the allocation and the issuer may either re-open Stage 1 for the remaining investors or proceed to Stage 2.

In Stage 2, the issuer decides on the price of shares and equally allocates them among the rest of the investors who is willing to accept the offered price. The investors before deciding whether or not to accept the offered price in Stage 2 may privately attempt to acquire information. After observing investors actions during Stage 1 and the reported signal (if any), the issuer decides on price at which to allocate shares during Stage 2.

In Stage 1, the issuer issues derivative contracts whose payoffs are contingent on the realization of the fundamental value v. Given that the firm value can take only two distinct values, it is sufficient to offer only two different contracts corresponding to each of the realization of the firm's value. The idea behind this is that an informed trader by choosing the specific derivative contract will reveal the information they possess about the realization. At the beginning of Stage 1, the issuer invites investors to apply for the allocation of the derivative contract on the "first come first served" basis (or alternatively, the issuer could randomly select an investor among those who applied). Application for Stage 1 trading is optional and each investor might choose not to apply and wait for Stage 2 allocation instead.

If an investor is selected to trade in round i of Stage 1, he has to pay the issuer a fixed fee F and then he gets the right to buy one contract of either C_H or C_L. The contract corresponding to the bad state C_L pays $F + f_{i,L}$ if $v = v_L$ at date 3 and zero otherwise; the contract corresponding to the good state C_H pays $F + f_{i,H}$ if $v = v_H$ and zero otherwise. The fee f_i is determined by the issuer before the issuance process, and depends in general on the information cost c, probability of successful information acquisition and the number of rounds in information acquisition.[7] It is designed to incentivize investors to acquire information (which is optional for them and they can choose not to pay information costs and not to acquire the information).

If the investor who has applied for Stage 1 and has been chosen to trade decides to report the neutral signal (either because he was unsuccessful in acquiring information or and he decided strategically to misreport and hide the information) does not trade any derivative contract and is excluded from the allocation. Exclusion of investors who refuse to trade in Stage 1 is needed to ensure the efficiency of the allocation and minimization of the cost of issuance. Suppose the investor receives a positive signal and strategically misreports the neutral signal betting on the issuer failing to acquire informative signal and offering the shares in Stage 2 at some average prices. In order to ensure truth telling and avoid this scenario the issuer has to offer higher compensation to those who disclose informative signal (by trading the derivative

[7]Each of these contracts can be replicated by issuing "butterfly spread" – a portfolio of call options written on the underlying asset, for example, C_L contract payoff is equivalent to the payoff of a long position in call option with strike price $f_{i,L} - F$, a short position in two call options with strike price v_L and a long position in a call option with strike price $f_{i,L} + F$ for given round i.

contract in Stage 1) which, in turn, increases the cost of issue. By excluding investors who declare neutral signal during Stage 1 eliminates this possibility as these investors will have no chance to exploit acquired information in the subsequent stage. On the other hand, investors who did not apply for Stage 1 trading can still participate in the following rounds of Stage 1 (should it have been announced) or being considered for allocation in Stage 2.

If during Stage 1 an investor bought one of the derivative contracts, the Stage 1 trading is ended and the issuer opens Stage 2 allocation. If the initial attempt to sell the derivative in Stage 1 fails, i.e., the investor who applied to trade derivatives decided not to close the trade (e.g., the signal received by the investor appeared to be neutral), then the issuer may call for the second round of application to trade in Stage 1. Any of the remaining investors (except those who have been excluded from the issue due to declaring neutral signal in one of the previous rounds) are allowed to participate. If Stage 1 results in unsuccessful trade after round K, the issuer terminates Stage 1 and proceed to Stage 2 allocation.

In Stage 2 the issuer allocates the Q shares among the remaining investors at price $p_{issue} = v_H$ if Stage 1 ends with the purchase of C_H contract, at price $p_{issue} = v_L$ if Stage 1 ends with the purchase of C_L contract, and at price $p_{issue} = \mu v_H + (1-\mu) v_L$ if Stage 1 ends with no transaction after $K_{\max}$ rounds. Each investor receives at most one share.

5.2 Issuer's objective and constraints

Similarly to our benchmark model, we define the issuer's problem as minimization of a separable function of accuracy of the price at Date 2, the expected amount of underpricing and expected cost of derivative trading (8). The main difference is that the issuer faces additional constraints relative to the benchmark model.

There are three main types of constraints that the optimal mechanism has to satisfy. The issuer needs to give investors the incentive both to buy the information and to report it accurately. As part of mechanism design problem, the issuer must design an allocation and pricing schedule that elicits accurate information from investors.

Since the issuer uses the reported information to price the issue, the pricing and allocation strategy must counteract investor incentives to withhold favorable information that will lead to a higher issue price. We will be considering Nash equilibria where, conditioned on the issuer's strategy, investors have an incentive to truthfully reveal their information, given their expectation that other investors will also report information accurately.

Let $R(s_i, \sigma)$ be the expected profit to an investor i who has been chosen to participate in the information acquisition process, receives signal s_i but decides to report the state σ instead (by means of choosing to trade the derivative C_σ, for $\sigma \in \{H, L\}$ or not to trade if $\sigma = U$). The assumption that the investors are excluded from the allocation when declaring neutral signal implies that $R(s_i, U) = 0$ for any $s_i \in \{H, L, U\}$. In equilibrium, investors are induced to report their information truthfully, which implies that the following truth-telling constraints must be satisfied:

$$R(s_i, s_i) \geq R(s_i, \sigma) \text{ for all } s_i, \sigma \in \{H, L, U\}. \tag{16}$$

It should be noted that the cost of acquiring information does not affect the information reporting conditions, since it is a sunk cost by the time the investor decides what signal to report. On the other hand, whether or not the investor plans to accurately report the signal certainly affects the incentive to buy a signal. After all, if the investor planned to report U (or H or L) regardless of the actual signal, then there would be no reason to buy a signal.

In addition to the truth-telling conditions, a constraint is needed to guarantee that investors choose to acquire information. The first set of conditions is that buying and reporting a signal offers at least as high an expected profit as not purchasing a signal and falsely reporting either H or L during Stage 1 trading:

$$\begin{aligned} \pi_i\phi(\mu R(H,H) + (1-\mu)R(L,L)) + (1-\pi_i\phi)R(U,U) \\ \geq R(\emptyset, \sigma) + c, \quad \sigma \in \{H, L, U\}, \quad i \leq \tau, \end{aligned} \tag{17}$$

where $R(\emptyset, \sigma)$ is the expected profit to an investor who reports σ without observing a signal. The profit of truthful reporting is equal to the profit from the corresponding derivative contract $R(s_i, s_i) = f_i(s_i)$ for $s_i \in \{H, L\}$ and the profit from reporting the neutral signal $R(U, U) = f_i(U) = 0$. The expected profit to an investor who reports σ without observing a signal is

$$R(\emptyset, \sigma) = \begin{cases} \mu(-F) + (1-p)f_{i,L}, & \sigma = L, \\ \mu f_{i,H} + (1-\mu)(-F), & \sigma = H. \end{cases} . \tag{18}$$

As a result, the condition (17) is equivalent to the following two conditions:

$$f_{i,H}(\pi_i \phi \mu) - f_{i,L}(1 - \pi_i \phi)(1-\mu) \geq c - \mu F, \tag{19}$$

$$f_{i,L}(\pi_i \phi (1-\mu)) - f_{i,H}(1 - \pi_i \phi)\mu \geq c - (1-\mu)F. \tag{20}$$

The second condition reflects the incentives of the investors to purchase a signal relative to patiently waiting for State 2 offering and not to apply for State 1 information acquisition:

$$\begin{aligned} \pi_i \phi(\mu R(H,H) + (1-\mu)R(L,L)) &+ (1-\pi_i\phi)R(U,U) - c \\ &\geq P(s=H)(v_H - p_{issue}(H)) + P(s=L)(v_L - p_{issue}(L)) \\ &+ P(s=U)R^{Stage2}(U), \quad i \leq \tau, \end{aligned} \tag{21}$$

where $R^{Stage2}(U)$ is the return that the investor receives from observing state U announced by the issuer in Stage 2. This return is either equal to $(\mu v_H + (1-\mu)v_L - p_{issue}(U))$ when the investor is allocated a share and accepts price $p_{issue}(U)$ or 0 if the investor is not allocated the share or refuses to participate.

5.3 Equilibrium

The optimal strategy of the investors (in terms of whether to acquire the information and whether to tell the truth) depends on their beliefs about issuer's commitment to start the Stage 2 even if it is unsuccessful in revealing the true fundamental value or not. Given that the information acquisition process can be lengthy and requires several rounds to obtain the information, we assume that there is no time discounting for the issuer. We discuss the implications of time discounting in the following sections.

The following analysis characterizes a Nash equilibrium in which each investor applies to Stage 1 trading, optimally pays for acquiring information and truthfully reveals the information to the issuer via purchasing the corresponding derivative contract.

Let us consider the state of the market when the investors failed to acquire information after i rounds ($s_i = U$ for all i). If the i traders who received those uninformative signals in the preceding i rounds genuinely tried to acquired information and did not hide informative signals, then the probability that there is additional information to be gained in the market conditional on observing i uninformative signals in a row is π_i, as defined in Equation (4).

Due to the fact that $\pi < 1$, investors might have incentives not to participate information acquisition and wait for Stage 2 betting that the information is not revealed during K rounds and try and acquire information privately. This happens, for example, when the number of rounds $K_{\max}$ is small (e.g., due to very low value of γ) and there is sufficiently high conditional probability of information acquisition π_i. In order to eliminate this possibility, the issuer has to invoke some additional mechanism that prevents information production. One example of such a mechanism can be **NI** described in the previous section.

The following proposition shows that the combination of **SEQ** and **NI** mechanisms implements the first-best allocation.

Proposition 5. *Consider the following mechanism:*

- *The issuer contacts each investor sequentially and asks them to produce information;*
- *The issuer stops contacting investor as soon as it obtains a positive ($s_i = H$) or a negative ($s_i = L$) report during round i or the number of rounds with unsuccessful reports exceeds $K_{\max}$;*

- *Conditional upon observing $s_i = U$, it sets $f_{i,L} = f_{i,H} = \varepsilon + \frac{c}{\pi_i \phi}$ with arbitrarily small $\varepsilon > 0$ and $F > \left(\varepsilon + \frac{c}{\phi \pi_{K_{\max}}}\right)$;*
- *If $s_i = H$ or $s_i = L$ for some $i \leq K_{\max}$ then the issuer sells Q shares to investors (chosen randomly) at price $p_{issue} = p_1(s_i)$.*
- *If $s = U$ for $i = K_{\max}$ then the issuer invokes* **NI** *mechanism and sells Q shares to investors at price $p_{issue} = E(v)$.*

Then each of I investors applies for Stage 1 trading and the expected utility of the issuer is given by

$$\begin{aligned}\Pi^*_{\mathbf{SEQ}} &= M + (Q+H)E(v) - cK_{\max}(1-\pi) - \frac{c\pi(1-(1-\phi)^{K_{\max}})}{\phi} \\ &- \gamma\mu(1-\mu)(1-\pi+\pi(1-\phi)^{K_{\max}})(v_H - v_L)^2.\end{aligned}$$

Proof. See Appendix. □

Corollary 6. FP *mechanism is never optimal.*

Proof. See Appendix. □

6 Discussion of results and limitations

The optimality of our mechanism is dependent on several key assumptions. In this section we discuss their relevance and limitations of the mechanism with respect to these assumptions.

One aspect where we are substantially different from Sherman and Titman (2002) as well as other papers in the literature is the sequential nature of our mechanism. In order to minimize the information acquisition costs, the issuer pays only one investor at each point in time rather than a number of investors at once. Optimality of our mechanism does not rely on the necessity of several investors having the same information to ensure truth-telling (as in Sherman and Titman, 2002). However, it might take several rounds for the investors to obtain information. In the model, the costs associated with a delay of the issue do not enter the objective function of the issuer and we assume that it is patient enough to wait as long as needed in order to produce the information (e.g., $E(\tau^*)$ increases as γ increases). If time were to enter the preferences of the issuer, the equilibrium solution would have to exhibit a trade-off between time preferences (speed of information acquisition) and its precision.

It should also be noted our model is that there is no secondary market for the derivatives. This means that the investor has to hold the derivative until maturity in order to cash out the reward for information production. Introducing a secondary market for derivatives is not straightforward as this might alter incentives of the investors for truth-telling in anticipation of potential derivative resale price.

Another important feature of our mechanism is the absence of an active market for shares before the derivative contracts trade (stage 1). This makes the IPO an ideal application of our mechanism. In the presence of an active parallel market (for example, SEO) the mechanism would still lead to the production of information and its full revelation in equilibrium. However, the availability of a market where the investor could trade after having acquired information would improve his outside option, maker his truth-telling constraint more binding and so increase the cost of information production for the issuer.

7 Conclusions

In this paper, we use a mechanism design approach to show that price informativeness can be achieved without illiquidity, at a cost equal to the information production cost. We build a model of stock issuance where the issuer incentivizes investors to search for costly information and truthfully disclose it. This is achieved by organizing the issue process in two stages where in the first stage, investors are sequentially offered the possibility to buy two derivatives securities, one that pays only if the asset payoff is high and one that pays only if it is low. We show that the entrepreneur can design the derivatives in such a way that an investor who participates to stage 1 finds it optimal to produce information and select the derivative security that truthfully reveals the asset payoff if she learned this payoff. Moreover, if an investor does not discover information, she optimally abstains from buying or selling a derivative. As a result, in the second stage, the issuer sells the asset at a price equal to its expected payoff.

The proposed two-stage mechanism allows the issuer to pay the information costs directly to the investor while efficiently relaxing incentive constraints (such as misreporting the information that investors obtain or not pay the cost of information acquisition). This, in turn, allows the issuer to avoid adverse selection costs.

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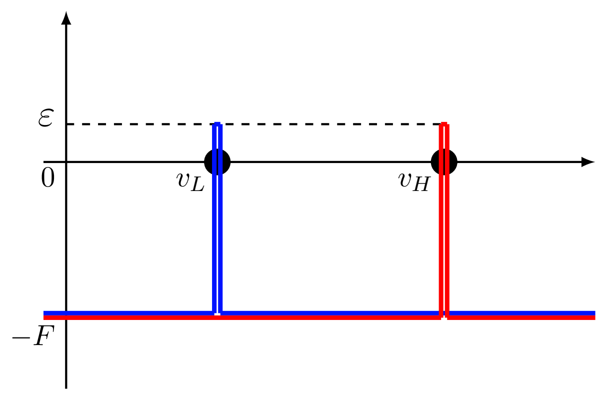
Appendix

Proof of Proposition 1. The mechanism must be incentive compatible both for informed and uninformed investors. Suppose that an informed investor applies to participate in Stage 1, observes the realization of the fundamental value v and chooses the derivative contract C_w, $w \in \{L, H\}$. The profit of the investor is:

$$\text{Profit}^{I}_{\text{Stage 1}}(w|v) = \begin{cases} \epsilon, & v = v_w, \\ -F, & v \neq v_w. \end{cases} \tag{22}$$

Figure below plots the profits for each of the contracts (the black line for C_L and the red line for C_H) as function of v.



The informed investor, upon participation in Stage 1, has incentive to disclose the information to the market maker via choosing the set of contracts corresponding to the true fundamental value. The profit of the informed in this case is $\text{Profit}^{I}_{\text{Stage 1}} = \epsilon > 0$.

Given a strictly positive profit in Stage 1, informed investors have incentives to participate in Stage 1 rather than Stage 2:

$$E\left[\text{Profit}^{I}_{\text{Stage 2}}\right] = v - p_{issue} = 0 < E\left[\text{Profit}^{I}_{\text{Stage 1}}\right].$$

The profit of an uninformed investor who decides to participate in stage 1 and chooses the contract C_w is

$$E\left[\text{Profit}^U_{\text{Stage }1}|w = v_L\right] = (1-\mu)\epsilon - \mu F < 0 = E\left[\text{Profit}^U_{\text{Stage }2}\right].$$

Hence, only informed investors choose to participate in the first stage, which leads to full information revelation. Finally, given that the value of ϵ is arbitrarily chosen, the loss of the issuer in this can be arbitrarily small. In a limit case when $\epsilon \to 0$, the loss of an issuer approaches to zero. □

Proof of Proposition 2. We first show that it is optimal to stop whenever $s_i = H$ or $s_i = L$. In this case, $s = s_i$ and since the information is revealed truthfully by assumption of the benchmark model, we have that $\mathrm{E}(Var(v \mid p_2)) = 0$. Furthermore,

$$\begin{aligned}
\Pi_i(p_{issue}, \{f_j\}) &= M + (N+Q)p_2(s_i) - \sum_{j=0}^{i} f_j(s_j) \\
&\geq M + (N+Q)p_2(s_i) - \sum_{j=0}^{k} f_j(s_j) = \Pi_k(p_{issue}, \{f_j\}).
\end{aligned}$$

for any $k > i$.

Next, we prove that $\tau^* \leq K_{\max}$. To do so, we first need to calculate the continuation value of the objective function at any round i for the next k number of rounds given that it is optimal to stop after the informative signal. First, note that

$$\mathrm{E}(Var(v \mid p_2)) = (1 - \pi_{i+1} + \pi_{i+1}(1-\phi)^k)\mu(1-\mu)(v_H - v_L)^2. \tag{23}$$

Indeed, if the process does not continue after k rounds, this means that the information revealed and $Var(v \mid p_2) = 0$. If this is not the case, then $s_{i+j} = U$ for any $j \leq k$ and the information is not revealed. Hence $Var(v \mid p_2) = \mu(1-\mu)(v_H - v_L)^2$. The latter case happens if there is either no information in the market (with probability $1 - \pi_{i+1}$) or the investors were unlucky to find one (with probability $\pi_{i+1}(1-\phi)^k$).

Next we calculate the expected fee needed to be paid for the next k rounds.

Lets denote by I_h the event that the issuer gets an informative signal exactly after contacting the hth investor (i.e., either $s_{i+h} = H$ or $s_{i+h} = L$ and all other previously contacted investors produces uncertain signal U). We also denote by U_h the event that the issuer gets uninformative signals from each investors $i+1, ..., i+h$ (i.e., $s_{i+j} = U$ for all $j \leq h$). In order to calculate the future expected fee (ignoring already paid sunk costs to the previous i investors) note that for any $h = 2, ..., k-i$:

$$\begin{aligned} E\left(\sum_{j=1}^{k} f_{i+j,s_{i+j}} | I_h\right) &= \sum_{j=1}^{h-1} f_{i+j,U} + \mu f_{i+h,H} + (1-\mu) f_{i+h,L} \equiv \sum_{j=1}^{h-1} f_{i+j,U} + \bar{f}_{i+h}, \\ Pr(I_h) &= \pi_{i+1}(1-\phi)^{h-1}\phi, \\ E\left(\sum_{j=1}^{k} f_{i+j,s_{i+j}} | U_k\right) &= \sum_{h=1}^{k} f_{i+h,U}, \\ Pr(U_k) &= 1 - \pi_{i+1} + \pi_{i+1}(1-\phi)^{i+k}. \end{aligned}$$

By the low of total expectations,

$$\begin{aligned} & E\left(\sum_{j=1}^{k} f_{i+j,s_{i+j}}\right) = \sum_{h=1}^{k} E\left(\sum_{j=1}^{k} f_{i+j,s_{i+j}} | I_h\right) Pr(I_h) + E\left(\sum_{j=1}^{k} f_{i+j,s_{i+j}} | U_k\right) Pr(U_k) \\ = \quad & \sum_{h=1}^{k} \left(\bar{f}_{i+h} + \sum_{j=1}^{h-1} f_{i+j,U}\right) \pi_{i+1}(1-\phi)^{h-1}\phi + \left(\sum_{h=1}^{k} f_{i+h,U}\right)\left(1 - \pi_{i+1} + \pi_{i+1}(1-\phi)^{i+k}\right) \\ = \quad & \sum_{h=1}^{k} \bar{f}_{i+h}\pi_{i+1}(1-\phi)^{h-1}\phi + \sum_{h=1}^{k} \pi_{i+1}(1-\phi)^{h-1}\phi \sum_{j=1}^{h-1} f_{i+j,U} \\ + \quad & \sum_{h=1}^{k} f_{i+h,U}\left(1 - \pi_{i+1} + \pi_{i+1}(1-\phi)^{i+k}\right) \\ \overset{Ineq.(10)}{\geq} \quad & \sum_{h=1}^{k} \frac{c\pi_{i+1}(1-\phi)^{h-1}}{\pi_{i+h}} - \sum_{h=1}^{k} f_{i+h,U}\left(\frac{1-\phi\pi_{i+h}}{\pi_{i+h}}\right)\pi_{i+1}(1-\phi)^{h-1} \\ + \quad & \sum_{h=1}^{k} \pi_{i+1}(1-\phi)^{h-1}\phi \sum_{j=1}^{h-1} f_{i+j,U} + \sum_{h=1}^{k} f_{i+h,U}\left(1 - \pi_{i+1} + \pi_{i+1}(1-\phi)^{i+k}\right) \end{aligned}$$

with the equality whenever the constraint (10) is binding.

Given that

$$\begin{aligned}
\frac{\pi_{i+1}(1-\phi)^{h-1}}{\pi_{i+h}} &= 1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{h-1}, && (24)\\
\left(\frac{1-\phi\pi_{i+h}}{\pi_{i+h}}\right)\pi_{i+1}(1-\phi)^{h-1} &= 1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{h}, && (25)\\
\sum_{h=1}^{k}(1-\phi)^{h-1}\sum_{j=1}^{h-1} f_{i+j,U} &= \sum_{h=0}^{k-1} f_{i+h,U}\frac{[(1-\phi)^h-(1-\phi)^k]}{\phi} && (26)
\end{aligned}$$

we have the following inequality:

$$\begin{aligned}
E\left(\sum_{j=1}^{k} f_{i+j,s_{i+j}}\right) &\geq cK(1-\pi_{i+1})+\frac{c\pi_{i+1}(1-(1-\phi)^k)}{\phi}-\sum_{h=1}^{k} f_{i+h,U}\left(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^h\right)\\
&+ \pi_{i+1}\sum_{h=0}^{k-1} f_{i+h,U}[(1-\phi)^h-(1-\phi)^k]+\sum_{h=1}^{k} f_{i+h,U}\left(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^k\right)\\
&= ck(1-\pi_{i+1})+\frac{c\pi_{i+1}(1-(1-\phi)^k)}{\phi}+\pi_{i+1}\sum_{h=0}^{k-1} f_{i+h,U}[(1-\phi)^h-(1-\phi)^k]\\
&- \pi_{i+1}\sum_{h=1}^{k} f_{i+h,U}[(1-\phi)^h-(1-\phi)^k] = ck(1-\pi_{i+1})+\frac{c\pi_{i+1}(1-(1-\phi)^k)}{\phi}.
\end{aligned}$$

Hence, the continuation value for up to k rounds is

$$\begin{aligned}
E\left(\Pi_k(p_{issue},\{f_i\})|\tau=i\right) &\leq M+(N+Q)\mathrm{E}(v)-ck(1-\pi_{i+1})-\frac{c\pi_{i+1}(1-(1-\phi)^k)}{\phi}\\
&- \gamma(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^k)\mu(1-\mu)(v_H-v_L)^2. \qquad (27)
\end{aligned}$$

We are ready to prove that it is sub-optimal to continue with the information search process if $\tau \geq K_{\max}$. We prove this by showing that the continuation value for any number of rounds $k>1$ is smaller than the expected value of the objective function $E\left(\Pi_0(p_{issue},\{f_i\})|\tau=i\right)$ when the process is stopped at τ.

Indeed, suppose that $\tau \geq K_{\max}$. Then

$$E\left(\Pi_0(p_{issue},\{f_i\})|\tau=i\right)=M+(N+Q)\mathrm{E}(v)-\gamma\mu(1-\mu)(v_H-v_L)^2.$$

Hence,

$$
\begin{aligned}
& E\left(\Pi_k(p_{issue},\{f_i\})|\tau=i\right) - E\left(\Pi_0(p_{issue},\{f_i\})|\tau=i\right) \\
\leq\ & \gamma\mu(1-\mu)(v_H-v_L)^2 \\
-\ & ck(1-\pi_{i+1}) - \frac{c\pi_{i+1}(1-(1-\phi)^k)}{\phi} - \gamma(1-\pi_{i+1}+\pi_{i+1}(1-\phi))\mu(1-\mu)(v_H-v_L)^2 \\
=\ & -ck(1-\pi_{i+1}) - \frac{c\pi_{i+1}(1-(1-\phi)^k)}{\phi} + \gamma\pi_{i+1}\left[1-(1-\phi)^k\right]\mu(1-\mu)(v_H-v_L)^2 \\
=\ & -ck(1-\pi_{i+1}) + \pi_{i+1}\left[1-(1-\phi)^k\right]\left[-\frac{c}{\phi}+\gamma\mu(1-\mu)(v_H-v_L)^2\right] \\
\leq\ & -ck(1-\pi_{i+1}) + \pi_{i+1}\left[1-(1-\phi)^k\right]\left[-\frac{c}{\phi}+\frac{c}{\phi\pi_{i+1}}\right] \\
=\ & -ck(1-\pi_{i+1}) + \frac{c(1-\pi_{i+1})\left[1-(1-\phi)^k\right]}{\phi} = c(1-\pi_{i+1})\left[-k+\frac{1-(1-\phi)^k}{\phi}\right] < 0.
\end{aligned}
$$

In order to finalize the proof we need to show that it is optimal to continue as long as $\tau^* \leq K_{\max}$. Suppose that the issuer managed to run $\tau = i$ information search rounds (with $0 < i < K_{\max}$) and all of them result in an uninformative signal U and

$$
\frac{c}{\pi_{i+1}\phi} < \mu(1-\mu)(v_H-v_L)^2. \tag{28}
$$

Then the expected cost of running at least one round of information search is less than or equals to

$$
\begin{aligned}
E\left(\Pi_1(p_{issue},\{f_i\})|\tau=i\right) &= M+(N+Q)\mathrm{E}(v) - \pi_{i+1}\phi\left[\mu f_{i+1,H}+(1-\mu f_{i+1,L})\right] \\
&- \gamma(1-\pi_{i+1}+\pi_{i+1}(1-\phi))\mu(1-\mu)(v_H-v_L)^2.
\end{aligned}
$$

According to the constraint (10), $\pi_{i+1}\phi\left[\mu f_{i,H}+(1-\mu f_{i,L})\right] \geq c$ but the issuer can achieve equality if it sets $\mu f_{i+1,H}+(1-\mu f_{i+1,L}) = \frac{c}{\pi_{i+1}\phi}$. Hence, the total expected cost in the case of one round of information search is

$$
E\left(\Pi_1(p_{issue},\{f_i\})|\tau=i\right) = M+(N+Q)\mathrm{E}(v)-c-\gamma(1-\pi_{i+1}+\pi_{i+1}(1-\phi)\mu(1-\mu)(v_H-v_L)^2.
$$

The difference in the expected objective functions is

$$
\begin{aligned}
& E\left(\Pi_1(p_{issue}, \{f_i\})|\tau = i\right) - E\left(\Pi_0(p_{issue}, \{f_i\})|\tau = i\right) \\
= \quad & \gamma\mu(1-\mu)(v_H - v_L)^2 - c - \gamma(1 - \pi_{i+1} + \pi_{i+1}(1-\phi))\mu(1-\mu)(v_H - v_L)^2 \\
= \quad & -c + \gamma\pi_{i+1}\phi\mu(1-\mu)(v_H - v_L)^2 > 0
\end{aligned}
$$

(the last inequality follows from inequality (28).

The choice $p_{issue} = p_1(s)$ is attainable and maximizes the objective function given the constraint (11). Furthermore, the ex-ante expected costs of the issuer is minimized when $\bar{f}_i \equiv \mu f_{i,H} + (1-\mu)f_{i,L} = \frac{c}{\phi\pi_i}$ and $f_{i,U} = 0$ and the expected objective function is equal to (13). □

Proof of Proposition 3. Let us suppose that the issuer decided to implement a hybrid procedure where it would call for M_i investors every round i who would search for the information simultaneously. The issuer promises to compensate them with fees $f_i(s_{i,m})$ depending on the signal they report, where $s_{i,m}$ is the signal reported by the investor m in round i. This compensation should satisfy for each m

$$
\pi_i\left[\phi\mu f_{i,H} + \phi(1-\mu)f_{i,L} + (1-\phi)f_{i,U}\right] + (1-\pi_i)f_{i,U} \geq c. \tag{29}
$$

So, as a result, the issuer's total expected fee in round i is

$$
\sum_{m=1}^{M_i}\{\pi_i[\phi\mu f_{i,H} + \phi(1-\mu)f_{i,L} + (1-\phi)f_{i,U}] + (1-\pi_i)f_{i,U}\} \geq M_i c. \tag{30}
$$

Suppose that the issuer selects the fee structure so that the equality holds in (30). The issuer's objective function for running k rounds of information search is

$$
\begin{aligned}
\Pi_k^{hybrid}(p_{issue}, \{f_i\}) \quad &= \quad M + (N+Q)\mathrm{E}(v) - E(C_{issue}) - \gamma E(Var(v \mid p_2)) \\
&= \quad M + (N+Q)\mathrm{E}(v) - cM_1 - Pr(i > 1)E(C_{future} \mid i > 1) \\
&- \quad \gamma(1 - \pi + \pi(1-\phi)^{M_1})E(Var(v \mid p_2) \mid i > 1),
\end{aligned}
$$

where $E(C_{future} \mid i > 1)$ is the expected future costs that the issue expected to incur conditional one more than one round going forward.

Consider now an alternative procedure, where instead of calling M_1 investors during the round 1 simultaneously, the issuer calls M_1 one by one to search for the information. If all of them fail to produce an informative signal, then the remaining procedure is identical to the initial hybrid one. Then the issuer's objective function for running those $M_1 + k - 1$ rounds (insuring that the same number of potential investors participates) of information search is this case is

$$\begin{aligned}\tilde{\Pi}^{hybrid}_{M_1+k-1}(p_{issue}, \{f_i\}) &= M + (N+Q)\mathrm{E}(v) - cM_1(1-\pi) - \frac{c\pi}{\phi}\left(1-(1-\phi)^{M_1}\right) \\ - \quad Pr(i > M_1)E(C_{future} \mid i > M_1) &- \gamma(1-\pi+\pi(1-\phi)^{M_1})E(Var(v \mid p_2) \mid i > M_1).\end{aligned}$$

Since, $M_1 > \frac{1-(1-\phi)^{M_1}}{\phi}$ we have that $\tilde{\Pi}^{hybrid}_{k}(p_{issue}, \{f_i\}) > \Pi^{hybrid}_{k}(p_{issue}, \{f_i\})$.

This means that no matter what M_1 the issuer chooses for the hybrid procedure, it is always better off in running M_1 sequential rounds first with the ex-ante pre-determined number of traders M_1 rather than calling them simultaneously. Given that the issue has flexibility of adjusting this M_1 ex-post (if the informative signal realizes sooner than M_1 rounds), this increases the expected objective function even further.

Finally, repeating this step for each round i with $M_i > 1$ shows that pure sequential procedure dominates the hybrid (or simultaneous) one. $\square$

Proof of Proposition 4. For the issue to succeed, the issuer must guarantee the participation of uninformed investors. Suppose that $v_L < p_{issue} < v_H$ and consider a situation in which it is optimal for each uninformed investor to buy one share at this price. At this price, each informed investor finds it optimal to buy one share if $v = v_H$ and to abstain otherwise. Thus, when $v = v_H$, each uninformed investor only receives $q_u(v_H) = \frac{Q}{H}$ shares (pro-rata rationing), while when $v = v_L$ each uninformed investor receives $q_u(v_L) = \frac{Q}{H(1-\lambda)}$. Thus, the expected profit of uninformed investors is:

$$E(q_u(v)(v - p_{issue})) = \mu q_u(v_H)(v_H - p_{issue}) + (1 - \mu)q_u(v_L)(v_L - p_{issue}).$$

To guarantee the participation of uninformed investors (which is necessary for the issue to succeed) and maximize the proceeds of the issue, the issuer must choose the largest price such that $E(q_u(v)(v - p_{issue})) \geq 0$, which is the price solving $E(q_u(v)(v - p_{issue})) = 0$. Thus, the issuing price is:

$$p^*_{issue} = \beta v_H + (1 - \beta)v_L,$$

with $\beta = \frac{\mu(1-\lambda)}{1-\mu\lambda}$. As $\lambda > 0$, we have: $p_{issue} < E(v)$. Thus, the issue must be underpriced for it to succeed. Note that in this case, the issuing price does not reveal information about v since it is identical whether informed investors participate or not in the issue. However, total demand in the issue fully reveals the asset payoff. Thus, if total demand is revealed ex-post, one obtains accuracy but at the cost of underpricing. This is a manifestation of the trade-off between illiquidity (here measured by underpricing) and informativeness.

To simplify, suppose that investors with the ability to produce the signal but who do not participate in the IPO (they will be indifferent in equilibrium). Likewise, suppose that informed investors who search for information but don't find it don't participate to the IPO. Now suppose that the price of the issue is such that $v_L < p_{issue} < v_H$. Thus, it is optimal for informed investors with information to demand one share when $v = v_H$ and to demand no shares when $v = v_L$. Moreover suppose that p_{issue} is such that it is optimal to buy one share for uninformed investors. Let $q_u(v)$ be the allocation to uninformed investors when the payoff of the asset is v and let $q_i(v)$ be the allocation to informed investors when the payoff of the asset is v. If $0 \leq k \leq K$ investors find information, we have:

1. $q_u(v_H) = q_i(v_H) = \frac{Q}{k+H}$
2. $q_u(v_L) = \frac{Q}{H}$, $q_i(v_L) = 0$

Note that k the number of informed participants in the IPO is random and that $Prob(k=j) = \pi\binom{K}{k}(1-\phi)^{K-k}(\phi)^k$ for $0 < k < K$ and $Prob(k=0) = \pi(1-\phi)^K + (1-\pi)$.

The expected profit of an uninformed investor when the price of the issue is p_{issue}:

$$\Pi_u(p_{issue}) = E(q_u(v)(v - p_{issue})). \tag{31}$$

The largest price of the issue that guarantees the participation of uninformed (which is necessary for the success of the issue when $v = v_L$) is therefore such that $\Pi_u(p_{issue}) = 0$, that is:

$$p_{issue} = \frac{E(q_u(v)v)}{E(q_u)}. \tag{32}$$

One can compute the price of the issue differently. Observe that the clearing condition in the IPO implies:

$$Hq_u(v) + kq_i(v) = Q \quad \text{for} \quad \forall k \text{ and } \forall v. \tag{33}$$

Thus,

$$H\Pi_u(p_{issue}) = E(Hq_u(v)(v - p_{issue})) = Q(E(v) - p_{issue}) - E(kq_i(v)(v - p_{issue})) = 0,$$

implying

$$Q(E(v) - p_{issue}) = E(kq_i(v)(v - p_{issue}). \tag{34}$$

This means that the total amount left on the table by the issuer is equal to informed investors' total expected profit. Moreover:

$$p_{issue} = \frac{E(v)Q}{Q - E(kq_i(v))} - \frac{E(kq_i(v)v)}{Q - E(kq_i(v))}. \tag{35}$$

Now, let $\tau(k) \equiv \frac{k}{k+H}$. $\tau(k)$ is the fraction of the issue allocated to informed investors when $v = v_H$. When $v = v_L$, informed investors do not trade. Thus, we have:

$$E(kq_i(v)v) = E(\tau(k))Q\mu v_H, \text{ and}$$

$$E(kq_i(v)) = E(\tau(k))Q\mu.$$

We deduce that

$$p_{issue} = \beta v_H + (1-\beta)v_L \tag{36}$$

with $\beta = \frac{\mu(1-E(\tau(k))}{1-E(\tau(k))\mu}$. Observe that $\beta < \mu$ if $E(\tau(k)) > 0$. Thus, informed trading in the IPO generates underpricing.

Given our assumptions, one can compute $E(\tau(k))$:

$$E(\tau(k)) = \pi \sum_{k=1}^{K} \binom{K}{k} (1-\phi)^{K-k} \phi^k \left(\frac{k}{k+H} \right). \tag{37}$$

Let $p^*_{issue}(K)$ be the equilibrium issue price when K investors search for information. In equilibrium, the aggregate expected profits of these investors is (from eq.(34)):

$$E(kq_i(v)(v - p^*_{issue}(K))) = Q(E(v) - p^*_{issue}(K)). \tag{38}$$

Thus, the aggregate expected profit of informed investors searching for information is equal to the expected loss of the issuer in the IPO (relative to an issue at the unconditional expected value of the asset).

Now consider the determination of K. Each informed investor who searches for information expects a profit of $\Pi_i(K) = \frac{E(kq_i(v)(v-p^*_{issue}(K)))}{K} = \frac{Q(E(v)-p^*_{issue}(K))}{K}$. As K increases, $\Pi_i(K)$ decreases (to be checked). And thus, K_{FP} is the largest value of K such that:

$$\Pi_i(K) \geq c. \tag{39}$$

Let K_{FP} be this value. We have:

$$K_{FP}\Pi_i(K_{FP}) \approx K^{FP}c. \tag{40}$$

In this approach, the aggregate demand in the IPO provides a more complex signal about the payoff of the asset. Let $D(v) = Q(q_i(v) + q_u(v)$ be this demand. It is either equal to H if $v = v_L$ or $v = v_H$ and $k = 0$ or strictly larger than H if $v = v_H$ and $k > 0$. Thus, when $D > H$, the IPO outcome reveals that $v = v_H$. If $D = H$, however, the

IPO demand is not fully revealing. Let $\mu(D = H, K) = Pr(v = v_H \mid D = H)$ when K investors search for information. We have:

$$\mu(D = H, K) = \frac{\mu(1 - \pi + \pi(1 - \phi)^K)}{\mu(1 - \pi + \pi(1 - \phi)^K) + (1 - \mu)} \tag{41}$$

Observe that $\mu(D = H, K) < \mu$. Observing that $D = H$ is bad news as it indicates the possibility that $v = v_L$. Note also that $\mu(D > H, K) = Pr(v = v_H \mid D > H) = 1$. It follows that:

$$\begin{aligned} E(Var(v \mid D)) &= \mu(D = H, K)(1 - \mu(D = H, K))(v_H - v_L)^2 \\ &= \frac{\mu(1 - \mu)(1 - \pi + \pi(1 - \phi)^K)}{\mu(1 - \pi + \pi(1 - \phi)^K) + (1 - \mu)}(v_H - v_L)^2. \end{aligned} \tag{42}$$

Thus, in equilibrium, the expected objective function of the issuer is:

$$\begin{aligned} \Pi_{FP} = M + NE(v) + QE(p^*_{issue}) - E(C_{issue}) - \gamma E(Var(v \mid p_2(s))) \\ \approx M + (Q + N)E(v) - cK_{FP} - \frac{\gamma\mu(1 - \mu)\left(1 - \pi + \pi(1 - \phi)^{K_{FP}}\right)(v_H - v_L)^2}{\mu\left(1 - \pi + \pi(1 - \phi)^{K_{FP}}\right) + (1 - \mu)}. \end{aligned} \tag{43}$$

□

Proof of Proposition 5. Let us start with verifying the truth-telling condition (16). Suppose that an investor applies to participate in Stage 1 and is chosen to acquire information. The investor observes the informative signal $s_i \in \{H, L\}$ and hence learns the realization of the true fundamental value v with probability ϕ. Conditional on observing the informative signal the investor purchases the corresponding derivative contract C_H if $s_i = H$ or C_L if $s_i = L$. The investor's profit is $R(H, H)$ or $R(L, L)$ respectively, and given that $f_{i,H} = f_{i,H} \equiv f_i$ is equal to:

$$R(H, H) = R(L, L) = F + \varepsilon + f_i - F = \varepsilon + \frac{c}{\phi\pi_i}.$$

Conditional on observing the neutral signal the trader is better off not participating in the trade as long as

$$R(U,H) = \mu\left(\varepsilon + \frac{c}{\phi\pi_i}\right) - (1-\mu)F < 0, \tag{44}$$

$$R(U,L) = (1-\mu)\left(\varepsilon + \frac{c}{\phi\pi_i}\right) - \mu F < 0. \tag{45}$$

Both inequalities (44) and (45) hold is we choose large enough F, that is, if

$$F > \max\left\{\frac{\mu}{1-\mu}, \frac{1-\mu}{\mu}\right\}\left(\varepsilon + \frac{c}{\phi\pi_{K_{\max}}}\right) \geq \max\left\{\frac{\mu}{1-\mu}, \frac{1-\mu}{\mu}\right\}\left(\varepsilon + \frac{c}{\phi\pi_i}\right).$$

Moreover, $R(H,L) = R(L,H) = -F < 0$. This verifies truth-telling constraint (16). Finally, since an investor reporting U signal is excluded from Stage 2 allocation, $R(H,U) = R(L,U) = -c$, and hence the investor has no incentive to sabotage and not to disclose an informative signal.

Next, we verify the set of conditions (19) and (20) that buying and reporting a signal offers at least as high expected profit as not purchasing a signal and falsely reporting either H or L during Stage 1 trading. Since

$$\left(\varepsilon + \frac{c}{\phi\pi_i}\right) \leq \left(\varepsilon + \frac{c}{\phi\pi_{K_{\max}}}\right) < \frac{F}{\max\left\{\frac{\mu}{1-\mu}, \frac{1-\mu}{\mu}\right\}},$$

the following relationship holds:

$$f_{i,H}(\phi\pi_i\mu) - f_{i,H}(1-\phi\pi_i)(1-\mu) = \left(\varepsilon + \frac{c}{\phi\pi_i}\right)(\pi_i\phi\mu) - \left(\varepsilon + \frac{c}{\phi\pi_i}\right)(1-\pi_i\phi)(1-\mu)$$
$$= \varepsilon\pi_i\phi + c - (1-\mu)\varepsilon - \frac{(1-\mu)c}{\phi\pi_i} > c - \left(\varepsilon + \frac{c}{\phi\pi_i}\right)(1-\mu)$$
$$> c - \frac{(1-\mu)F}{\max\left\{\frac{\mu}{1-\mu}, \frac{1-\mu}{\mu}\right\}} \geq c - \mu F,$$

which proves condition (19). Similarly,

$$f_{i,L}(\pi_i\phi(1-\mu)) - f_{i,H}(1-\pi_i\phi)\mu = \left(\varepsilon + \frac{c}{\phi\pi_i}\right)(\pi_i\phi(1-\mu)) - \left(\varepsilon + \frac{c}{\phi\pi_i}\right)(1-\pi_i\phi)\mu$$

$$= \varepsilon\pi_i\phi(1-\mu) + c - \mu\varepsilon - \frac{\mu c}{\phi\pi_i} > c - \left(\varepsilon + \frac{c}{\phi\pi_i}\right)\mu$$

$$> c - \frac{\mu F}{\max\left\{\frac{\mu}{1-\mu}, \frac{1-\mu}{\mu}\right\}} \geq c - (1-\mu)F,$$

which proves that the condition (20) holds.

To verify condition (21) that the investors have incentives to purchase a signal relative to patiently waiting for State 2 offering, we note that $R^{Stage2}(U) = 0$. Indeed, after the issuer invokes the **NIP** mechanism, the investor has no incentives to acquire information privately is better off accepting price $p_{issue}(U) = \mu v_H + (1-\mu)v_L$, in which case $R^{Stage2}(U) = 0$. Hence, condition (21) follows from this argument.

As a result, all additional constraints are satisfied, and since the optimal stopping rule, and choice of the functions $\{f_i\}$ and p_issue are identical to the benchmark model, the allocation achieved in this mechanism coincides with the benchmark allocation. Hence, the expected value of the objective function is equal to

$$\begin{aligned} \Pi^*_{\mathbf{SEQ}} &= M + (Q+N)E(v) - cK_{\max}(1-\pi) + \frac{c\pi(1-(1-\phi)^K_{\max})}{\phi} \\ &- \gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{\max}}\right)(v_H - v_L)^2 \end{aligned} \quad (46)$$

$\square$

Proof of Corollary 6. Consider the following two cases: a) $K_{FP} \leq K_{\max}$ and b) $K_{FP} > K_{\max}$.

a). Let us modify the sequential mechanism so that the we bound the stopping time from above by $\tau \leq K_{FP}$. Let us denote the expected objective function of the issuer in this case by

$$\Pi_{\mathbf{SEQ}}(\tau \le F_{FP}) = \max_{\{p_{issue},\{f_i\}\}} \Pi(p_{issue}, \{f_i\}, \tau \le F_{FP}),$$

where $\Pi(p_{issue}, \{f_i\}, \tau \le F_{FP})$ is the value of the optimal stopping problem

$$\Pi(p_{issue}, \{f_i\}, \tau \le F_{FP}) = \sup_{0\le\tau\le F_{FP}} \Pi_\tau(p_{issue}, \{f_i\}),$$

Following the proof of Propositions 2 and 5 we can deduce that

$$\begin{aligned}\Pi^*_{\mathbf{SEQ}}(\tau \le F_{FP}) &= M + (Q+N)E(v) - cK_{FP}(1-\pi) + \frac{c\pi(1-(1-\phi)^{K_{FP}})}{\phi} \\ &- \gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{FP}}\right)(v_H - v_L)^2 < \Pi^*_{\mathbf{SEQ}}.\end{aligned}$$

Hence, we have

$$\begin{aligned}& \Pi_{FP} - \Pi_{SEQ} < \Pi_{FP} - \Pi_{SEQ}(\tau \le F_{FP}) \\ = \;& cK_{FP}(1-\pi) + \frac{c\pi(1-(1-\phi)^{K_{FP}})}{\phi} + \gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{FP}}\right)(v_H-v_L)^2 \\ - \;& cK^{FP} - \frac{\gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K^{FP}}\right)(v_H-v_L)^2}{\mu\left(1-\pi+\pi(1-\phi)^{K^{FP}}\right)+(1-\mu)} \\ < \;& -cK_{FP}\pi + \frac{c\pi(1-(1-\phi)^{K_{FP}})}{\phi} \\ + \;& \gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{FP}}\right)(v_H-v_L)^2 - \gamma\mu(1-\mu)\left(1-\pi+\pi(1-\phi)^{K_{FP}}\right)(v_H-v_L)^2 \\ = \;& \frac{c\pi}{\phi}\left(1-(1-\phi)^{K_{FP}} - K_{FP}\phi\right) < 0. \qquad (47)\end{aligned}$$

b). Consider a round i such that $i > K_{\max}$, so we have

$$c > \pi_{i+1}\phi\gamma\mu(1-\mu)(v_H - v_L)^2. \qquad (48)$$

In this case it is optimal for **SEQ** mechanism to stop. Let us show that in this case **NIP** mechanism also dominates **FP** mechanism. Indeed, note that **NIP** mechanism is preferred over **FP** mechanism by the issuer if and only if:

$$U \equiv cK_{FP} - \frac{\gamma\pi_{i+1}\mu(1-\mu)^2(v_H - v_L)^2(1-(1-\phi)^{K_{FP}})}{\mu(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{K_{FP}}) + (1-\mu)} > 0. \tag{49}$$

Since (48) holds, we can write:

$$\begin{aligned}
U &> K_{FP}\pi_{i+1}\phi\gamma\mu(1-\mu)(v_H - v_L)^2 - \frac{\gamma\pi_{i+1}\mu(1-\mu)^2(v_H - v_L)^2(1-(1-\phi)^{K_{FP}})}{\mu(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{K_{FP}}) + (1-\mu)} \\
&= \pi_{i+1}\gamma\mu(1-\mu)(v_H - v_L)^2\left(K_{FP}\phi - \frac{(1-\mu)(1-(1-\phi)^{K_{FP}})}{\mu(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{K_{FP}}) + (1-\mu)}\right) \\
&> \pi_{i+1}\gamma\mu(1-\mu)(v_H - v_L)^2\left((1-(1-\phi)^{K_{FP}}) - \frac{(1-\mu)(1-(1-\phi)^{K_{FP}})}{\mu(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{K_{FP}}) + (1-\mu)}\right) \\
&= \pi_{i+1}\gamma\mu(1-\mu)(v_H - v_L)^2(1-(1-\phi)^{K_{FP}})\left(1 - \frac{1-\mu}{\mu(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{K_{FP}}) + (1-\mu)}\right) \\
&= \pi_{i+1}\gamma\mu(1-\mu)(v_H - v_L)^2(1-(1-\phi)^{K_{FP}})\left(1 - \frac{1-\mu}{\mu(1-\pi_{i+1}+\pi_{i+1}(1-\phi)^{K_{FP}}) + (1-\mu)}\right) \\
&> \pi_{i+1}\gamma\mu(1-\mu)(v_H - v_L)^2(1-(1-\phi)^{K_{FP}})\left(1 - \frac{1-\mu}{\mu(1-\pi_{i+1}) + (1-\mu)}\right) > 0
\end{aligned}$$

This shows that whenever it is optimal to stop within **SEQ** mechanism, it is not optimal to invoke **FP** as an alternative.

Suppose now that $i > K_{\max}$. We can show that **SEQ** mechanism dominates **FP** in the similar way as in a). To do so, we can modify **SEQ** mechanism by forcing the issuer to continue the information search until round $K_{FP} > K_{\max}$ if $s_i \neq U$ for $i = K_{\max+1}, ..., K_{FP}$. The difference in the expected objective functions are then

$$\Pi_{FP} - \Pi_{SEQ} < \Pi_{FP} - \Pi_{SEQ}(K_{\max} \leq \tau \leq F_{FP}) < 0.$$

following the same logic as in (47). $\square$